

Selection Examination for the Training Program

Mathematics

1 Answer the following questions.

(1) Find the limit

$$\lim_{n \rightarrow \infty} \frac{\sqrt{n+1} - \sqrt{n}}{\sqrt{n+2} - \sqrt{n-1}}.$$

(2) Compute the integral

$$\int_0^1 x^5(1-x)^3 dx.$$

(3) Arrange the following three numbers in ascending order:

$$2^{1/2}, \quad e^{1/e}, \quad 3^{1/3}.$$

2 Consider the functions $z = z(x)$ and $w = w(x)$ defined by

$$z = e^{2x} \cos x, \quad w = e^{2x} \sin x.$$

Answer the following questions. Notation: $z' = dz/dx$, $w'' = d^2w/dx^2$, etc.

(1) Find four numbers $\alpha, \beta, \gamma, \delta$ such that

$$z' = \alpha z + \beta w, \quad w' = \gamma z + \delta w.$$

(2) Show that z and w satisfy

$$z'' - 4z' + 5z = 0, \quad w'' - 4w' + 5w = 0.$$

(3) Find a function $y = y(x)$ that satisfy

$$y'' - 4y' + 5y = 0, \quad y(0) = 2, \quad y'(0) = 1.$$

3 Let $A = \begin{pmatrix} 0 & 1 \\ -6 & 5 \end{pmatrix}$. Answer the following questions.

(1) Compute

$$A \begin{pmatrix} 1 \\ 2 \end{pmatrix} \quad \text{and} \quad A \begin{pmatrix} 1 \\ 3 \end{pmatrix}.$$

(2) Find two numbers α, β such that

$$\alpha \begin{pmatrix} 1 \\ 2 \end{pmatrix} + \beta \begin{pmatrix} 1 \\ 3 \end{pmatrix} = \begin{pmatrix} 8 \\ 11 \end{pmatrix}.$$

(3) Put

$$\begin{pmatrix} x_n \\ y_n \end{pmatrix} = A^n \begin{pmatrix} 8 \\ 11 \end{pmatrix}, \quad n = 1, 2, 3, \dots$$

Find the limit

$$\lim_{n \rightarrow \infty} \frac{x_n}{y_n}.$$

4 Answer the following questions.

(1) Let $A = \begin{pmatrix} 1 & 3 \\ 1 & 2 \\ 1 & 1 \end{pmatrix}$ and $B = \begin{pmatrix} 1 & 1 & 2 \\ 2 & 1 & 1 \end{pmatrix}$. Compute

$$AB \quad \text{and} \quad \det(AB).$$

(2) Compute

$$\det \begin{pmatrix} 1 & 1 & 1 & 1 \\ 1 & 2 & 3 & 4 \\ 1 & 3 & 6 & 10 \\ 1 & 4 & 10 & 20 \end{pmatrix}.$$

(3) Compute the angle between the two vectors

$$u = \begin{pmatrix} 1 \\ -2 \\ 2 \end{pmatrix}, \quad v = \begin{pmatrix} 3 \\ 4 \\ -5 \end{pmatrix}.$$