Selection Examination for the Training Program

[for secondary recruitment and third year transfer]

Mathematics

- There are FOUR questions in total.
- Be sure to attempt the ALL questions.
- Examinees have 2 hours.
- No examinee is allowed to leave the examination rooms during the first 1 hours and the last 10 minutes.
- Books, note books, dictionaries, mobile phones, and information devices must be kept in the specified place.

[Important Notes]

1 : Do not turn this page until instructed to do so by the proctor.
2 : There are 6 pages including this cover page. Please make sure you are not missing a page after the exam begins.
3 : Be sure to write your name and examinee's number on all the answer sheets.
4 : Use one answer sheet per question. Be sure to write the question number on the sheet.
5 : Write answers in English or Japanese.
6 : You may use the reverse side of answer sheet as calculation sheet.
7 : If you need to use two or more sheets for one problem, be sure to mention at the end of the answer sheet that the answer continues on the next sheet.
8 : When you need an additional answer sheet, notify the proctor by raising your hand, and receive a new answer sheet.
9 : After the exam finishes, sort the answer sheets numerically, by question number, and fold in half.
10 : You may take this booklet with you after the exam finishes.

THIS PAGE CONTAINS NO QUESTIONS.

$\boxed{\mathbf{1}}$ Answer the following questions.

(i) Find all matrices X which satisfy the condition that
$$X = \begin{pmatrix} a & b \\ b & c \end{pmatrix}, \text{ where } a, b, c \text{ are integers, and } X^2 = \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}.$$
Do not use the symbol "$\pm$" (plus-minus sign) or "$\mp$" (minus-plus sign) in your answer. Integers are the numbers $0, 1, -1, 2, -2, 3, -3$, and so forth.

(ii) Find the rank of the matrix
$$A(a) = \begin{pmatrix} 1 & 1 & 1 \\ a & 1 & 1 \\ 1 & a^2 & 1 \end{pmatrix}$$
for every real number a.

(iii) Compute the determinant of the matrix
$$\begin{pmatrix} 0 & 1 & 2 & 1 \\ 1 & 1 & 0 & 0 \\ 2 & 0 & -1 & 0 \\ 1 & 0 & 0 & -1 \end{pmatrix}.$$

$\boxed{\mathbf{2}}$ Let f be the function defined by
$$f(x) = \int_0^{\pi} (x-t)^2 \sin t \, dt \quad (-\infty < x < \infty).$$
Find the minimum of f.

3 Let $\{x_n\}_{n=0}^{\infty}$ and $\{y_n\}_{n=0}^{\infty}$ be sequences defined by the recurrence formula

$$\begin{cases} x_{n+1} = 5x_n + 4y_n & (n = 0, 1, 2, \dots), \\ y_{n+1} = 2x_n + 7y_n & (n = 0, 1, 2, \dots), \end{cases}$$

with initial values $x_0 = 4$, $y_0 = 1$. Answer the following questions.

(i) Find a matrix A with which the recurrence formula above is written as

$$\begin{pmatrix} x_{n+1} \\ y_{n+1} \end{pmatrix} = A \begin{pmatrix} x_n \\ y_n \end{pmatrix}.$$

(ii) Let $P = \begin{pmatrix} 1 & -2 \\ 1 & 1 \end{pmatrix}$. Compute $P^{-1}AP$.

(iii) Find explicit expressions for x_n and y_n.

4 Let $f(x, y, z)$ be the function defined by

$$f(x, y, z) = \frac{1}{\sqrt{x^2 + y^2 + z^2}} \quad \text{for} \quad (x, y, z) \neq (0, 0, 0).$$

Answer the following questions.

(i) Compute

$$\frac{\partial^2 f}{\partial x^2} + \frac{\partial^2 f}{\partial y^2} + \frac{\partial^2 f}{\partial z^2}.$$

(ii) Compute

$$\iiint_{x^2+y^2+z^2 \leq 1} f(x, y, z)\, dxdydz.$$

THIS PAGE CONTAINS NO QUESTIONS.

プログラム履修者選抜試験問題

[**2** 次募集および **3** 年次編入]

数学

- この問題冊子には問題が 4 問 ある.
- すべての問題 を解答せよ.
- 解答時間は 2 時間 である.
- 試験開始後 1 時間から 1 時間 50 分の間 は途中退席してもよい.
- 参考書・ノート類・辞書・携帯電話・情報機器等は，指定された荷物置場に置くこと.

[注意]

1: 指示のあるまで問題冊子を開かないこと.
2: この問題冊子は 6 ページ ある．試験開始後に全てのページがあるか確認すること.
3: 解答用紙のすべてに氏名・受験番号を記入すること.
4: 解答用紙は問題ごとに別の用紙を用い，それぞれ解答した問題番号を記入すること.
5: 解答は日本語または英語のどちらでもよい.
6: 解答用紙の裏面は計算用紙として使ってよい.
7: 一問を二枚以上にわたって解答するときは，つづきのあることを用紙下端に明示して次の用紙に移ること.
8: 解答用紙が足りなくなったら挙手により試験監督を呼び，新しい解答用紙を受け取ること.
9: 提出の際は解答用紙を問題番号順に重ね，記入した面を外にし，一括して二つ折りにして提出すること.
10: この問題冊子は持ち帰ってもよい.